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Modeling Insurance Loss Data: The Log-EIG Distribution

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Modeling Insurance Loss Data: The Log-EIG Distribution

Uditha Balasooriya,* Chan Kee Low,† and Adrian Y.W. Wong‡

Abstract§

The log-EIG distribution was recently introduced to the probability literature. It has positive support and a moderately long tail, and is closer to the lognormal than to the gamma or Weibull distributions. Our simulations show that data generated from a log-EIG distribution cannot be adequately described by lognormal, gamma, or Weibull distributions. The log-EIG distribution is a worthwhile candidate for modeling insurance claims (loss) data or lifetime data. Examples of fitting the log-EIG to published insurance claims data are given.

Key words and phrases: *claims distribution, optimal invariant selection procedure, Akaike information criterion, simulation, fitting distributions*

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1 Introduction

In fitting distributions to insurance loss data, several families of distributions have been proposed. The common characteristics of these distributions are their skewness to the right and their long tails to capture occasional large values that are commonly present in insurance loss data. One fundamental question confronting actuaries, reliability analysts, and other researchers, however, is the approach used to select the best model for a given data set.

Various approaches have been proposed for discriminating between families of distributions. For example:

- Lehmann (1959) has provided the so-called most powerful invariant test, which is uniformly most powerful in the class of tests that are invariant under certain transformations of the data.

- There is the separate families test based on the Neyman-Pearson maximum likelihood ratio; see, for example, Cox (1962). The concept of separate families of distributions is important, as it is natural to consider competing families in model selection.

- Geisser and Eddy (1979) have proposed a synthesis of Bayesian and sample-reuse approach for model selection. The emphasis here is to obtain a model that yields the best prediction for future observations.

- The maximum likelihood ratio test was proposed by Dumonceaux, Antle, and Haas (1973) for selecting between two models with unknown location and scale parameters. This test has the advantage that the distribution of the ratio of the two likelihood functions does not depend on the location and scale parameters. Gupta and Kundu (2003) used this test to discriminate between Weibull and generalized exponential distributions.

- Marshall, Meza, and Olkin (2001) used maximum likelihood and Kolmogorov distance methods to compare selected lifetime distributions, including the gamma, Weibull, and lognormal.

- Quesenberry and Kent (2001) proposed a method for selecting between distributions based on statistics that are invariant under scale transformation of the data. As pointed out by Quesenberry and Kent, however, for selecting among distributions that involve both shape and scale parameters, an optimal invariant procedure does not always exist.

- Selection based on the goodness-of-fit test, such as Pearson chi-square and the Kolmogorov-Smirnov tests, often results in more than one family of distributions deemed to be fitting the data well. This approach therefore does not always lead to selecting the best distribution for a given set of data.

In a recent paper, Guiahi (2001) discussed the issues and methodologies for fitting alternative parametric probability distributions to samples of insurance loss data. When exact sizes of loss are available, Scollnik (2001 and 2002) discussed how the Bayesian inference software package WinBUGS can be used to model loss distributions. Cairns (2000) provides detail discussion on parameter and model uncertainty.

The degree of difficulty in discriminating between two distributions has been explained by Littell, McClave, and Often (1979) and Bain and Engelhardt (1980). The problem is that often more than one family of distributions may exhibit a good fit to a given set of data. Bain and Engelhardt have pointed out that even though two models may offer similar degree of fit to a data set (even for moderate sample sizes), it is still desirable to select the correct (or more nearly correct) model, if possible, because inferences based on the model will often involve tail probabilities where the effect of the model assumption will be more critical.

The concept of long-tailed (sometimes called "heavy-tailed") distribution conveys the idea of relatively large probability mass at extreme values of the random variable. In the literature, it seems that what constitutes a long-tailed distribution depends on the context of the problem at hand and the distributions that are compared. For example, in analyzing time-varying volatility of financial data, long-tailed distributions are described as having kurtosis measure larger than the normal distribution (see Campbell, Lo, and MacKinlay 1997, pp. 480–481).

In ruin theory, heavy-tailed distributions are sometimes defined as those that satisfy the Cramér-Lundberg theorem for the probability of ultimate ruin (see Embrechts, Klüppelberg and Mikosch 1997, p. 43). One approach to compare the tail behavior of two arbitrary density functions, $f(x)$, $g(x)$, is to examine the ratio $f(x)/g(x)$ as x tends to infinity. If $g(x)$ has a heavier (lighter) tail than $f(x)$, then the ratio approaches zero (infinity) as x tends to infinity; see, for example, Klugman, Panjer, and Willmot (2004, Chapter 4.3).

In loss modeling, the concern is usually with the tail of the distribution. Small losses do not cause as much concern as large ones, so it is important that the fitted distribution has sufficient probability mass in the tail to adequately capture the probability of large losses. This is particularly relevant in reinsurance where one is required to price a high-excess layer. For this reason, in practice the lognormal and Weibull distributions are more often used than the gamma distribution.

The objective of this paper is to investigate the performance of a new model, called the log-EIG distribution, proposed by Saw, Balasooriya, and Tan (2002) and to compare it with other commonly used distributions for fitting insurance losses and other applications. It appears that the log-EIG has some features that are somewhat different from the other commonly used distributions such as the gamma, lognormal, and Weibull. In this regard, the log-EIG distribution, which generally has a thicker tail than both the Weibull and gamma distributions, is a good candidate for modeling loss data. In selecting among competing distributions, we employ the Quesenberry and Kent (2001) selection criterion. Using a Monte Carlo simulation study, we investigate the usefulness of the log-EIG distribution and its features. We also illustrate the practical usefulness of this distribution through applications to three published insurance data sets. For two of these data sets, we show that the log-EIG fits the data best, when compared with the lognormal, gamma, and Weibull distributions.

2 Properties of the Log-EIG

Saw, Balasooriya, and Tan (2002) introduced the log-EIG as an alternative loss distribution with non-zero coefficient of skewness. Its probability density function (pdf) is given by

$$\begin{aligned}\mathrm{LEIG}(x,\theta_1,\theta_2) = {} & \frac{1}{\sqrt{2\pi}\theta_2 x}\left(\frac{\theta_1}{x}\right)^{1/(2\theta_2)} \\ & \times \exp\left[-2\left(\sinh\left(\frac{1}{2\theta_2}\ln\frac{x}{\theta_1}\right)\right)^2\right]\end{aligned} \tag{1}$$

for $x > 0$, where $\theta_i > 0$ for $i = 1,2$; θ_1 is a scale parameter and θ_2 is a shape parameter. The cumulative distribution function (cdf) of the log-EIG takes the form

$$\begin{aligned}F_X(x) = {} & \Phi\left(\left(\frac{x}{\theta_1}\right)^{1/2\theta_2} - \left(\frac{\theta_1}{x}\right)^{1/2\theta_2}\right) \\ & + e^2\Phi\left(-\left(\frac{x}{\theta_1}\right)^{1/2\theta_2} - \left(\frac{\theta_1}{x}\right)^{1/2\theta_2}\right)\end{aligned} \tag{2}$$

where, as usual, $\Phi(\cdot)$ denotes the standard normal cdf. The mean and variance of the log-EIG distribution are

$$\text{Mean} = c\theta_1 K_{\theta_2 - \frac{1}{2}}(1) \tag{3}$$

$$\text{Variance} = c\theta_1^2 \left[K_{2\theta_2 - \frac{1}{2}}(1) - cK^2_{\theta_2 - \frac{1}{2}}(1) \right] \tag{4}$$

where $c = e\sqrt{\frac{2}{\pi}}$, and

$$K_{\theta_2 k - \frac{1}{2}}(1) = \int_0^\infty \frac{1}{2} w^{\theta_2 k - \frac{3}{2}} \exp\left\{ -\frac{(w + w^{-1})}{2} \right\} dw$$

is a modified Bessel function; see, for example, Zhang and Jin (1996). For convenience, the probability density functions of the gamma, lognormal, and Weibull together with their means and variances are given below: the gamma distribution with parameters α and γ has pdf

$$G(x, \alpha, \gamma) = \frac{x^{\alpha - 1}}{\gamma^\alpha \Gamma(\alpha)} \exp(-\frac{x}{\gamma}),$$

with mean $\alpha\gamma$ and variance $\alpha\gamma^2$; the Weibull distribution with parameters λ and β has pdf

$$W(x, \lambda, \beta) = \frac{\beta}{\lambda} \left(\frac{x}{\lambda} \right)^{\beta - 1} \exp\left[-\left(\frac{x}{\lambda} \right)^\beta \right],$$

with mean $\lambda\Gamma\left(1 + \frac{1}{\beta}\right)$ and variance $\lambda^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right) \right]$, and the lognormal distribution with parameters μ and σ has pdf

$$\text{LN}(x, \mu, \sigma) = \frac{1}{\sigma x \sqrt{2\pi}} \exp\left\{ -\frac{[\ln(x/\mu)]^2}{2\sigma^2} \right\}$$

with mean $\mu \exp\left(\frac{\sigma^2}{2}\right)$ and variance $\mu^2[\exp(2\sigma^2) - \exp(\sigma^2)]$.

One can use the ratio of the density functions to show that the lognormal has a heavier tail than the gamma distribution, and that the log-EIG has a heavier tail than the gamma. For the case of Weibull, the ratio of the log-EIG pdf to the Weibull pdf is

$$\exp\left[\left(\frac{x}{\lambda} \right)^\beta - \frac{1}{2} \left(\frac{x}{\theta_1} \right)^{\frac{1}{\theta_2}} - \left(\frac{1}{2\theta_2} + \beta \right) \ln x \right].$$

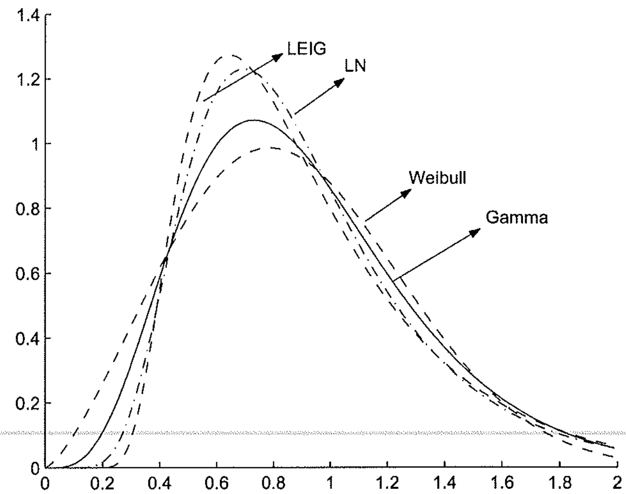

Figure 1: PDFs with Mean = 0.913149 and Variance = 0.166158

When $\beta > 1/\theta_2$ the above ratio approaches infinity when $x \to \infty$. Therefore, the log-EIG has a heavier tail than Weibull when $\beta > 1/\theta_2$.

The pdf of the gamma, log-EIG, lognormal and Weibull corresponding to a common mean and variance equal to 0.91315 and 0.16616, respectively, are shown in Figure 1. Notice that the log-EIG has the highest peak and they are all skewed to the right. Closeness of the log-EIG curve to the lognormal curve is clearly evident from Figure 1.

The functional form of the hazard function for log-EIG is analytically intractable. Saw, Balasooriya, and Tan (2002) have plotted the hazard function for several parameter values and show that it is generally non-monotone. Nevertheless, depending on the parameter values, the log-EIG distribution can accommodate a variety of situations corresponding to monotonic as well as non-monotonic failure rates.

Two important attributes of claim distributions are (i) the limited expected value (LIMEV) and (ii) the layered expected value (LAYEV). The limited expected value of a claim amount random variable X is

$$\mathrm{LIMEV}_X(u) = \mathbb{E}[min(X, u)],$$

where u is the policy limit. In Table 1 we compare the LIMEV of the log-EIG, lognormal, gamma, and Weibull corresponding to u equal to the

Table 1
Limited Expected Values of Distributions with Fixed Mean and Variance at Selected Percentiles of the Log-EIG Distribution

	LEIG	LN	G	W
	Mean = 0.91315 and Variance = 0.16616			
	$\theta_1 = 1.0$	$\mu = 0.8338$	$\alpha = 5.0184$	$\lambda = 1.0302$
u (%tile)	$\theta_2 = 0.5$	$\sigma = 0.4263$	$\gamma = 0.1820$	$\beta = 2.3846$
1.1154 (P_{75})	0.7372	0.8249	0.8244	0.8259
1.7094 (P_{95})	0.8228	0.8967	0.9014	0.9065
2.2325 (P_{99})	0.8597	0.9092	0.9117	0.9129
	Mean = 1.0 and Variance = 1.0			
	$\theta_1 = 1.0$	$\mu = 0.7071$	$\alpha = 1.0$	$\lambda = 1.0$
u (%tile)	$\theta_2 = 1.0$	$\sigma = 0.8326$	$\gamma = 1.0$	$\beta = 1.0$
1.2441 (P_{75})	0.7342	0.7482	0.7118	0.7118
2.9221 (P_{95})	0.9353	0.9374	0.9462	0.9462
4.9841 (P_{99})	0.9855	0.9822	0.9932	0.9932
	Mean = 2.0 and Variance = 33.0			
	$\theta_1 = 1.0$	$\mu = 0.6576$	$\alpha = 0.1212$	$\lambda = 0.6955$
u (%tile)	$\theta_2 = 2.0$	$\sigma = 1.4915$	$\gamma = 16.500$	$\beta = 0.4226$
1.5477 (P_{75})	1.1818	0.7968	0.4548	0.6073
8.5385 (P_{95})	1.9833	1.5455	1.2852	1.4046
24.8412 (P_{99})	2.0000	1.8395	1.8248	1.8157

Notes: %tile = Percentile and $P_\epsilon = \epsilon^{\text{th}}$ percentile.

$75^{\text{th}}, 95^{\text{th}}$, and 99^{th} percentiles of the log-EIG when $\theta_1 = 1.0$, and $\theta_2 = 0.5, 1.0$, and 2.0. The parameter values of the competing distributions are chosen to give the same mean and variance of the log-EIG. When $\theta_1 = 1.0$ and $\theta_2 = 0.5$, the log-EIG has the smallest LIMEV among the competing distributions, whereas, when $\theta_1 = 1.0$ and $\theta_2 = 2.0$, it has the largest LIMEV. This seems to indicate that the tail thickness of the log-EIG is sensitive to changes in θ_2 values.

The layered expected claim, on the other hand, is the expected claims corresponding to different layers of insurance. Knowledge of the layered expectation is useful to insurers and reinsurers when pricing policies with deductibles and retention limits. If X is the incurred loss on a policy with a deductible L_d and a retention limit L_u, the claim amount Y paid by the insurer is given by

$$Y = \begin{cases} 0 & \text{if } X \le L_d \\ X - L_d & \text{if } L_d < X \le L_u \\ L_u - L_d & \text{if } X > L_u. \end{cases}$$

The layered expected claim is $\text{LAYEV}(L_d, L_u) = \mathbb{E}(Y)$, i.e.,

$$\text{LAYEV}(L_d, L_u) = \int_{L_d}^{L_u} (x - L_d)\mathrm{d}F_X(x) + (L_u - L_d)\int_{L_d}^{\infty} \mathrm{d}F_X(x),$$

where $F_X(x)$ is the cdf of X. The above equation can be expressed as

$$\text{LAYEV}(L_d, L_u) = \text{LIMEV}(L_u) - \text{LIMEV}(L_d).$$

In addition, the average amount per payment, AAPP, is given by:

$$\text{AAPP} = \frac{\text{LIMEV}(L_u) - \text{LIMEV}(L_d)}{P(X > L_d)}.$$

As the AAPP and $\text{LAYEV}(L_d, L_u)$ for the log-EIG are analytically complex, in Table 3 we present the AAPP and $\text{LAYEV}(L_d, L_u)$ for the competing distributions for selected L_d and L_u values corresponding to the $5^{\text{th}}, 75^{\text{th}}, 95^{\text{th}}$, and 99^{th} percentiles of the log-EIG distribution. We note from the tabulated values that the log-EIG is distinctly different from the other distributions for all the cases considered. This further indicates that the log-EIG represents a family of distributions which exhibit significant differences to the more commonly used lognormal, gamma, and Weibull distributions.

Saw, Balasooriya, and Tan (2002) have discussed the maximum likelihood estimation of the log-EIG parameters, which involves the solution of two nonlinear equations. As there are no closed-form solutions, numerical methods such as the Newton-Raphson[1] have to be used to obtain the maximum likelihood estimates.

In the case of grouped data, as is common for insurance loss data, maximum likelihood estimation may proceed along the same line as discussed in Hogg (1984, p. 122). Again, iterative methods are required to obtain maximum likelihood estimates. Alternatively, one could use other methods such as the minimum distance or minimum chi-square, as discussed in Hogg (1984, pp. 143–151).

[1] For more on the numerical solution of nonlinear equations see, for example, Burden and Faires (2001, Chapter 2).

Table 2

Average Amount per Payment and $\mathbb{E}(Y)$ for Selected Layers of the Loss Distributions with Fixed Mean and Variance

		LEIG	LN	G	W
		Mean = 0.91315 and Variance = 0.16616			
		$\theta_1 = 1.0$	$\mu = 0.8338$	$\alpha = 5.0184$	$\lambda = 1.0302$
L_d †	L_u †	$\theta_2 = 0.5$	$\sigma = 0.4263$	$\gamma = 0.1820$	$\beta = 2.3846$
0.4291 (P_{05})	1.1154 (P_{75})	0.8348(0.2746)‡	0.5264(0.4951)	0.4433(0.4038)	0.4663(0.4119)
1.154(P_{75})	1.7094(P_{95})	0.8733(0.0856)	0.3646(0.0090)	0.2842(0.0077)	0.2701(0.0081)
1.7094 (P_{95})	2.2325 (P_{99})	0.3347(0.0154)	0.9522(0.2368)	0.2368(0.0104)	0.1807(0.0064)
		Mean = 1.0 and Variance = 1.0			
		$\theta_1 = 1.0$	$\mu = 0.7071$	$\alpha = 1.0$	$\lambda = 1.0$
L_d	L_u	$\theta_2 = 1.0$	$\sigma = 0.8326$	$\gamma = 1.0$	$\beta = 1.0$
0.1841 (P_{05})	1.2441 (P_{75})	1.2604(0.5521)	0.7310(0.6923)	0.6535(0.5437)	0.6535(0.5437)
1.2441 (P_{75})	2.9221 (P_{95})	1.7442(0.2010)	0.9522(0.2368)	0.8133(0.2344)	0.8133(0.2344)
2.9221 (P_{95})	4.9841 (P_{99})	2.1796(0.0502)	1.2519(0.0553)	0.8728(0.0470)	0.8728(0.0470)
		Mean = 2.0 and Variance = 33.0			
		$\theta_1 = 1.0$	$\mu = 0.6576$	$\alpha = 0.1212$	$\lambda = 0.6955$
L_d	L_u	$\theta_2 = 2.0$	$\sigma = 1.4915$	$\gamma = 16.500$	$\beta = 0.4226$
0.0339 (P_{05})	1.5477 (P_{75})	2.4897(1.1479)	0.9235(0.9019)	0.8735(0.4361)	0.7659(0.5794)
1.5477 (P_{75})	8.5385 (P_{95})	3.6231(0.8015)	3.3025(0.9347)	3.9169(0.8303)	3.2402(0.7973)
8.5385 (P_{95})	24.8412 (P_{99})	4.0259(0.0167)	8.5380(0.3656)	7.8038(0.5396)	7.3645(0.4111)

Notes: † Values in parentheses are percentiles of the LEIG distribution; ‡ Values in parentheses are $\mathbb{E}(Y)$.

Table 3

Percentage of Selections Among Different Groups of Candidate Models Using the QK Criterion when n = 50 and 100†

Model	Number of Candidate Models						
	4				2	2	2
LEIG	LEIG	LN	G	W	LN	G	W
$\theta_2 = 0.5$	74.95	14.95	9.05	1.05	22.80	14.10	3.85
	75.78	*20.42*	*3.80*	*0.00*	*23.92*	*8.31*	*0.50*
$\theta_2 = 1.0$	86.55	9.15	3.90	0.40	12.65	5.35	3.10
	85.40	*13.70*	*0.90*	*0.00*	*14.60*	*1.40*	*0.60*
$\theta_2 = 2.0$	75.55	21.85	0.00	2.60	23.80	1.85	4.50
	78.40	*21.30*	*0.00*	*0.30*	*21.60*	*0.30*	*0.60*
LN					LEIG	G	W
$\sigma = 0.5$	35.95	41.25	19.75	3.05	36.85	23.50	7.85
	27.60	*55.30*	*16.80*	*3.00*	*27.80*	*17.30*	*2.60*
$\sigma = 1.0$	31.90	56.35	10.70	1.05	48.75	11.95	9.95
	35.70	*60.50*	*3.80*	*0.00*	*35.70*	*3.80*	*2.00*
$\sigma = 2.0$	39.40	51.60	0.15	8.85	39.85	2.70	9.20
	35.20	*62.70*	*0.00*	*2.10*	*35.20*	*0.30*	*2.10*
G					LEIG	LN	W
$\gamma = 0.5$	1.45	0.75	68.30	29.50	3.05	4.35	31.60
	0.00	*0.00*	*75.00*	*25.00*	*0.00*	*0.30*	*25.00*
$\gamma = 1.0$	3.15	6.35	45.00	45.50	5.85	9.75	49.35
	0.00	*1.40*	*52.60*	*46.00*	*0.00*	*1.80*	*46.70*
$\gamma = 2.0$	11.85	8.80	47.95	31.40	15.80	18.90	31.40
	0.30	*8.40*	*64.50*	*26.80*	*1.50*	*8.70*	*26.80*
W					LEIG	LN	G
$\beta = 0.5$	6.25	5.20	23.20	65.35	8.85	10.65	23.20
	0.00	*3.00*	*15.70*	*81.30*	*1.10*	*3.00*	*15.70*
$\beta = 1.0$	2.75	6.30	47.20	43.75	5.85	9.95	51.95
	0.30	*2.70*	*49.70*	*47.30*	*1.60*	*3.10*	*51.60*
$\beta = 2.0$	1.75	2.50	25.40	70.35	6.55	10.30	29.65
	0.10	*0.00*	*18.50*	*81.40*	*0.70*	*2.80*	*18.60*

Notes: Italicized values refer to n = 100.

Table 3 (Contd.)
Percentage of Selections Among Different Groups of Candidate Models Using the QK Criterion when $n = 50$ and $n = 100$

Model	Number of Candidate Models								
	3			3			3		
LEIG	LEIG	LN	G	LEIG	LN	W	LEIG	G	W
$\theta_2 = 0.5$	74.95	14.95	10.10	76.85	20.85	2.30	85.90	13.05	1.05
	75.78	*20.42*	*3.80*	*76.08*	*23.82*	*0.10*	*91.69*	*8.31*	*0.00*
$\theta_2 = 1.0$	86.55	9.15	4.30	87.15	10.70	2.15	94.65	4.95	0.40
	85.40	*13.70*	*0.90*	*85.40*	*14.30*	*0.30*	*98.60*	*1.40*	*0.00*
$\theta_2 = 2.0$	76.15	22.80	1.05	75.55	21.85	2.60	95.50	0.00	4.50
	78.40	*21.50*	*0.10*	*78.40*	*21.30*	*0.30*	*99.40*	*0.00*	*0.60*
LN	LEIG	LN	G	LEIG	LN	W	LN	G	W
$\sigma = 0.5$	35.95	41.25	22.80	36.75	55.50	7.75	76.50	20.45	3.05
	27.60	*55.30*	*17.10*	*27.80*	*69.60*	*2.60*	*65.70*	*34.00*	*0.30*
$\sigma = 1.0$	31.90	56.35	11.75	32.00	58.05	9.95	88.05	10.09	1.05
	35.70	*60.50*	*3.80*	*35.70*	*62.30*	*2.00*	*96.20*	*3.80*	*0.00*
$\sigma = 2.0$	39.70	57.65	2.65	39.40	51.60	9.00	90.80	0.15	9.05
	35.20	*64.50*	*0.30*	*35.20*	*62.70*	*2.10*	*97.90*	*0.00*	*2.10*

Notes: Italicized values refer to $n = 100$.

Table 3 (Contd.)

Percentage of Selections Among Different Groups of Candidate Models Using the QK Criterion when n = 50 and n = 100

Model	Number of Candidate Models								
	3			3			3		
G	LEIG	LN	G	LEIG	G	W	LN	G	W
$\gamma = 0.5$	1.95	2.65	95.40	1.95	68.35	29.70	1.85	68.35	29.80
	0.00	*0.30*	*99.70*	*0.00*	*75.00*	*25.00*	*0.00*	*75.00*	*25.00*
$\gamma = 1.0$	3.15	6.80	90.05	5.60	46.90	47.50	9.30	45.20	45.50
	0.00	*1.80*	*98.20*	*0.00*	*53.30*	*46.70*	*1.40*	*52.60*	*46.00*
$\gamma = 2.0$	11.85	8.80	79.35	15.80	52.80	31.40	18.90	49.70	31.40
	0.30	*8.40*	*91.30*	*1.50*	*71.70*	*26.80*	*8.70*	*64.50*	*26.80*
W	LEIG	LN	W	LEIG	G	W	LN	G	W
$\beta = 0.5$	6.25	5.20	88.55	8.85	23.20	67.95	10.65	23.20	66.15
	0.00	*3.00*	*97.00*	*1.10*	*15.70*	*83.20*	*3.00*	*15.70*	*81.30*
$\beta = 1.0$	2.90	7.35	89.75	5.20	49.05	45.75	8.85	47.30	43.85
	0.30	*2.80*	*96.90*	*1.50*	*50.50*	*48.00*	*3.00*	*49.70*	*47.30*
$\beta = 2.0$	2.05	8.30	89.65	2.75	26.90	70.35	4.00	25.65	70.35
	0.10	*2.70*	*97.20*	*0.10*	*18.50*	*81.40*	*0.10*	*18.50*	*81.40*

Notes: Italicized values refer to n = 100.

3 Selection Procedure

For a given set of n observations $x_1, x_2, \cdots, x_n$, suppose it is required to choose one member from among a set of competing families of distributions $F_1, F_2, \cdots, F_k$ with scale and shape parameters, ϑ_i and ν_i, that best fits the data. Let f_i be the probability density function corresponding to F_i, $i = 1, 2, \ldots k$. The optimum invariant selection criterion of Quesenberry and Kent (2001) selects F_i which maximizes the selection statistic

$$S_i = \int_0^\infty f_i(tx_1, tx_2, \cdots, tx_n)\; t^{n-1} dt,$$

where $\vartheta_i = 1, i = 1, 2, \ldots, k$. Note, for a random sample $x_1, x_2, \ldots, x_n$, the above function can be expressed as a product of the f_i's, i.e.,

$$f_i(tx_1, tx_2, \cdots, tx_n) = \prod_{j=1}^{n} f_i(tx_j).$$

For the case of log-EIG where $\vartheta_i = \theta_1 = 1$ and $\nu_i = \theta_2$, it can be shown that the statistic, S_i, is given by

$$\left(\frac{e}{\sqrt{2\pi}\theta_2}\right)^n \prod_{j=1}^{n} \left(\frac{1}{x_j}\right)^{1+1/2\theta_2} \int_0^\infty \frac{1}{t^{1+n/2\theta_2}} \exp\left\{-\frac{1}{2}\left(t^{1/\theta_2}\phi + \frac{\psi}{t^{1/\theta_2}}\right)\right\} dt,$$

where $\phi = \sum_{j=1}^{n} x_j^{1/\theta_2}$ and $\psi = \sum_{j=1}^{n} x_j^{-1/\theta_2}$. The selection statistics for the other distributions can be similarly derived and are given in Quesenberry and Kent (2001).

When $\nu_1, \nu_2, \ldots, \nu_k$ are unknown, Quesenberry and Kent (2001) proposed that a suitable scale invariant estimate be substituted for ν_i. The selection criterion is then said to be suboptimal invariant. From extensive Monte Carlo studies involving the gamma, lognormal, and Weibull distributions, Quesenberry and Kent (2001) established that the proposed selection procedure performs well when selecting among families of distributions with shape and scale parameters.

For the log-EIG, lognormal, and Weibull distributions, when applying the suboptimal procedure, we substitute the shape parameter by its maximum likelihood estimates in the computation of S_i. Following Quesenberry and Kent (2001), for the gamma distribution we employ the approximate maximum likelihood estimate of the shape parameter proposed by Greenwood and Durand (1960); that is

$$\hat{\nu} = \begin{cases} \dfrac{0.5000876 + 0.1648852R - 0.0544274R^2}{R} & \text{for } 0 < R \leq 0.5772, \\ \dfrac{8.898919 + 9.059950R + 0.9775373R^2}{R(17.79728 + 11.968477R + R^2)} & \text{for } 0.5772 < R \leq 17, \end{cases}$$

where

$$R = \ln\left(\frac{\text{arithmetic mean of the observations}}{\text{geometric mean of the observations}}\right).$$

In selecting among probability models one also can use information theoretic criteria such as the Akaike information criterion (AIC) or some of its modifications such as the AIC with finite corrections (AICC) [Sugiura, 1978], or the Bayesian information criterion (BIC) [Schwarz, 1978]. For the four distributions considered in this paper, the AIC, AICC, and BIC give identical results because these distributions have the same dimension.[2] Thus, for comparing with the Quesenberry and Kent criterion (QK), we only report the selection results using the AIC criterion.

4 Simulation Results

In our simulation study, we generated 2,000 random samples of size $n = 50$ and 1,000 samples of size $n = 100$ from each of the four distributions gamma, log-EIG, lognormal, and Weibull. Random observations from the lognormal, gamma, and Weibull distributions were generated using MATLAB® standard routines for selected values of the parameters. For the log-EIG distribution, random observations were obtained by first generating inverse Gaussian variates using Dataplot and then transforming them to log-EIG variates using the relationships between the inverse Gaussian, exponential inverse Gaussian, and the log-EIG distributions; see Kanefuji and Iwase (1996) and Saw et al. (2002). It follows from these relationships that if Z is distributed as Inverse Gaussian with shape and location parameters both equal to 1, then $X = \theta_1 Z^{\theta_2}$ has a LEIG(θ_1, θ_2) distribution.

Table 3 presents percentages of selections among different groupings of candidate models consisting of 4, 3, and 2 competing distributions when the data are generated by the model indicated in the first

[2]When the competing models have the same number of parameters, they are said to have the same dimension; see Judge, Griffiths, Hill, Lütkepohl, and Lee (1985, pp. 870–873).

column of the table. The values in parentheses are percentages of selections when $n = 100$. For example, the entries 74.95, 14.95, 9.05, 1.05 at the beginning of the table mean that when the data are generated from a log-EIG distribution with parameters $\theta_1 = 1$ and $\theta_2 = 0.5$, the suboptimal selection procedure selected the log-EIG, lognormal, gamma, and Weibull as the population distribution 74.95%, 14.95%, 9.05%, and 1.05% of the time, respectively. The tabulated values under the heading '3' give the percentages of selections for groups of three competing distributions where the true population distribution is one of the competing members. The tabulated values under the heading '2' give the percentages of selections for the specified distribution under each heading, when compared with the population distribution indicated in the first column of the table. The entries therefore represent percentages of incorrect selections. For comparison, in Table 4 we present percentages of correct selection using the AIC selection criterion.

In distinguishing the log-EIG when it is the true population with all the alternative groupings of families considered, the lowest percentage of the correct selection is 74.95 (73.35) for the case when $\theta_2 = 0.5$ ($\theta_2 = 2.0$). To save space, note that throughout this section the figures in parentheses refer to the corresponding values for AIC criterion reported in Table 4. When data are generated from the lognormal, gamma, and Weibull distributions, the lowest percentage of correct selections are 41.25% (28.80%) when $\sigma = 0.5 (\sigma = 0.5)$, 45.00% (42.15%) when $\gamma = 1.0 (\gamma = 1.0)$ and 43.75% (46.50%) when $\beta = 1.0 (\beta = 1.0)$, respectively. This seems to indicate that the log-EIG, the new addition to the location and scale family of distributions, has some features that are somewhat different from the other commonly used loss distributions.

From the tabulated values in Tables 3 and 4, we note that when the true distribution is log-EIG, among the other competing three distributions, the lognormal is selected more often than the gamma or Weibull. On the other hand, when the true distribution is lognormal, the log-EIG is selected more often than the gamma or Weibull in all the groupings considered. For example, when two distributions compete, and samples of size $n = 50$ are generated from lognormal with $\sigma = 0.5, 1.0, 2.0$, log-EIG is selected 36.85% (50.0%), 48.75% (48.75%), 39.85% (44.40%) versus 23.50% (24.60%), 11.95% (12.95%), 2.70% (3.75%) for G, and 7.85% (8.60%), 9.95% (10.75%), 9.20% (10.35%) for Weibull, respectively. The corresponding figures for lognormal when the samples are generated from log-EIG with $\theta_2 = 0.5, 1.0, 2.0$ are 22.80% (23.15%), 12.65% (23.50%), 23.80% (26.45%), versus 14.10% (14.25%), 5.35% (7.70%), 1.85% (2.70%) for G, and 3.85% (4.65%), 3.10% (4.30%), 4.50% (5.00%) for Weibull, respectively. The same pattern is observed for the case of $n = 100$ al-though the corresponding percentages of incorrect for log-EIG and lognormal are somewhat lower than when $n = 50$. These findings seem to indicate that the log-EIG is closer to the lognormal than to the gamma or Weibull distributions.

While both QK and AIC criteria yield high percentages of correction selections, the QK performs marginally better in most of the cases considered in this simulation study. The QK criterion, however, is computationally more involved than the AIC.

Next we consider the situation when data arise from a log-EIG distribution but the investigator considers choosing one of the gamma, lognormal or Weibull to fit the data. Table 5 gives the percentages of selections for gamma, lognormal, and Weibull by the suboptimal selection procedure for the competing groupings $\{G, \text{LN}, \text{Weibull}\}$, $\{G, \text{LN}\}$, $\{\text{LN}, \text{Weibull}\}$, and $\{G, \text{Weibull}\}$ when the data are generated from the log-EIG with various values of the shape parameter θ_2. Again as we observed earlier, the tabulated values clearly indicate that the lognormal distribution is the closest distribution to the log-EIG for all the θ_2 values considered. When only gamma and Weibull are considered, gamma appears to be closer to log-EIG for $\theta_2 = 0.5$ or 1.0, while Weibull is closer to log-EIG when $\theta_2 = 2.0$. This is consistent with the higher selection proportions for gamma when $\theta_2 = 0.5$ or 1.0 and higher selection proportion for Weibull when $\theta_2 = 2.0$ in the simulation results reported in Tables 3 and 4. Therefore, it seems that when gamma and Weibull compete to represent log-EIG, the selection depends on the shape parameter of the log-EIG from which the data arise.

The similarities/differences among the four distributions are further illustrated by Table 6 which compares selected percentile values of the distributions with the same mean and variance, i.e., given the first two moments of the distributions. The selected common means and variances correspond to the log-EIG when $(\theta_1, \theta_2) = (1.0, 0.5)$, (1.0, 1.0), (1.0, 2.0). The parameter values for the lognormal, gamma, and Weibull distributions for the given means and variances are reported in the table. From the table, it can be seen that the percentiles for lognormal are closer to that of the log-EIG than to the gamma or Weibull. Further, the percentiles for gamma are closer to the log-EIG than the Weibull for $(\theta_1, \theta_2) = (1.0, 0.5)$, (1.0, 1.0), while the converse is true when $(\theta_1, \theta_2) = (1.0, 2.0)$. These observations are consistent with the simulation results reported in Tables 3, 4, and 5 and provide some theoretical justification for the simulation results.

Table 4
Percentage of Selections Among Different Groups of Candidate Models Using the AIC Criterion when $n = 50$ and $100^{\dagger}$

Model	Number of Candidate Models						
	4				2	2	2
LEIG	LEIG	LN	G	W	LN	G	W
$\theta_2 = 0.5$	75.20	14.05	9.55	1.20	23.15	14.25	4.65
	79.10	*16.70*	*4.20*	*0.00*	*20.40*	*8.40*	*0.50*
$\theta_2 = 1.0$	75.95	19.00	4.45	0.60	23.50	7.70	4.30
	80.30	*18.70*	*1.00*	*0.00*	*19.60*	*1.80*	*0.60*
$\theta_2 = 2.0$	73.35	23.20	0.05	3.40	26.45	2.70	5.00
	77.20	*22.40*	*0.00*	*0.40*	*22.80*	*0.30*	*0.90*
LN					LEIG	G	W
$\sigma = 0.5$	48.80	28.80	18.95	3.45	50.00	24.60	8.60
	34.40	*48.40*	*16.80*	*0.40*	*34.90*	*17.40*	*2.70*
$\sigma = 1.0$	48.35	39.15	11.10	1.40	48.75	12.95	10.75
	36.40	*59.70*	*3.90*	*0.00*	*36.40*	*3.90*	*2.30*
$\sigma = 2.0$	44.20	45.75	0.25	9.80	44.40	3.75	10.35
	39.30	*58.40*	*0.00*	*2.30*	*39.30*	*0.30*	*2.30*
G					LEIG	LN	W
$\gamma = 0.5$	0.85	1.05	72.10	26.00	2.30	3.80	27.80
	0.00	*0.00*	*77.40*	*22.60*	*0.00*	*0.30*	*22.60*
$\gamma = 1.0$	3.05	5.60	42.15	49.20	6.15	8.90	52.55
	0.10	*1.20*	*51.70*	*47.00*	*0.60*	*1.40*	*47.60*
$\gamma = 2.0$	7.50	10.45	46.65	35.40	12.35	17.65	35.40
	1.10	*7.20*	*62.30*	*29.40*	*3.50*	*8.70*	*29.40*
W					LEIG	LN	G
$\beta = 0.5$	3.65	6.55	26.65	63.15	6.45	9.90	26.65
	0.30	*2.50*	*16.80*	*80.40*	*1.30*	*2.80*	*16.80*
$\beta = 1.0$	3.10	5.40	45.00	46.50	5.95	9.30	49.60
	0.00	*2.80*	*47.50*	*49.70*	*1.50*	*2.90*	*49.30*
$\beta = 2.0$	2.50	1.80	22.55	73.15	6.55	9.65	26.85
	0.10	*0.00*	*17.40*	*82.50*	*0.90*	*2.70*	*17.50*

Notes: Italicized values refer to $n = 100$.

Table 4 (Contd.)
Percentage of Selections Among Different Groups of
Candidate Models Using the AIC Criterion when n = 50 and n = 100

	Number of Candidate Models								
Model	3			3			3		
LEIG	LEIG	LN	G	LEIG	LN	W	LEIG	G	W
$\theta_2 = 0.5$	75.20	14.05	10.75	76.45	20.80	2.75	85.75	13.05	1.20
	79.10	*16.70*	*4.20*	*79.60*	*20.20*	*91.60*	*91.60*	*8.40*	*0.00*
$\theta_2 = 1.0$	75.95	19.00	5.05	76.20	21.10	2.70	92.30	7.10	0.60
	80.30	*18.70*	*1.00*	*80.40*	*19.30*	*0.30*	*98.20*	*1.80*	*0.00*
$\theta_2 = 2.0$	73.45	25.05	1.50	73.35	23.20	3.45	95.00	0.05	4.95
	77.20	*22.70*	*0.10*	*77.20*	*22.40*	*0.40*	*99.10*	*0.00*	*0.90*
LN	LEIG	LN	G	LEIG	LN	W	LN	G	W
$\sigma = 0.5$	48.80	28.80	22.40	49.65	42.05	8.30	75.40	21.10	3.50
	34.40	*48.40*	*17.20*	*34.90*	*62.40*	*2.70*	*82.60*	*17.00*	*0.40*
$\sigma = 1.0$	48.35	39.15	12.50	48.45	40.95	10.60	87.05	11.55	1.40
	36.40	*59.70*	*3.90*	*36.40*	*61.30*	*2.30*	*96.10*	*3.90*	*0.00*
$\sigma = 2.0$	44.35	51.95	3.70	44.20	45.75	10.05	89.65	0.25	10.10
	39.30	*60.40*	*0.30*	*39.30*	*58.40*	*2.30*	*97.70*	*0.00*	*2.30*

Notes: Italicized values refer to n = 100.

Table 4 (Contd.)

Percentage of Selections Among Different Groups of Candidate Models Using the AIC Criterion when $n = 50$ and $n = 100$

Model	3			3			3		
G	LEIG	LN	G	LEIG	G	W	LN	G	W
$\gamma = 0.5$	0.95	2.90	96.15	1.20	72.10	26.70	1.75	72.15	26.10
	0.00	*0.30*	*99.70*	*0.00*	*77.40*	*22.60*	*0.00*	*77.40*	*22.60*
$\gamma = 1.0$	3.20	5.90	90.90	5.80	43.75	50.45	8.55	42.25	49.20
	0.10	*1.30*	*98.60*	*0.60*	*52.00*	*47.40*	*1.30*	*51.70*	*47.00*
$\gamma = 2.0$	7.50	10.45	82.05	12.35	52.25	35.40	17.65	46.95	35.40
	1.10	*7.20*	*91.70*	*3.50*	*67.10*	*29.40*	*8.30*	*62.30*	*29.40*
W	LEIG	LN	W	LEIG	G	W	LN	G	W
$\beta = 0.5$	3.65	6.55	89.80	6.45	26.65	66.90	9.90	26.65	63.45
	0.30	*2.50*	*97.20*	*1.30*	*16.80*	*81.90*	*2.80*	*16.80*	*80.40*
$\beta = 1.0$	3.10	6.45	90.45	5.45	46.75	47.80	8.20	45.10	46.70
	0.00	*2.90*	*97.10*	*1.30*	*48.40*	*50.30*	*2.80*	*47.50*	*49.70*
$\beta = 2.0$	2.95	6.75	96.30	3.15	23.70	73.15	3.80	23.05	73.15
	0.10	*2.60*	*97.30*	*0.10*	*17.40*	*82.50*	*0.10*	*17.40*	*82.50*

Notes: Italicized values refer to $n = 100$.

Table 5
Percentage of Selections, Using the QK Criterion, Among Different Groups of Candidate Models in the Absence of LEIG When Data are Generated from Log-EIG for $n = 50$ and 100

	Number of Candidate Models									
Model	3			2		2		2		
LEIG	LN	G	W	LN	G	LN	W	G	W	
$\theta_2 = 0.5$	87.25	11.70	1.05	87.25	12.75	97.50	2.50	98.95	1.05	
	95.80	*4.20*	*0.00*	*95.80*	*4.20*	*99.90*	*0.10*	*100.0*	*0.00*	
$\theta_2 = 1.0$	95.05	4.55	0.44	95.05	4.95	97.60	2.40	98.15	1.85	
	98.90	*1.10*	*0.0*	*98.9*	*1.10*	*99.70*	*0.30*	*99.90*	*0.10*	
$\theta_2 = 2.0$	97.00	0.00	3.00	98.80	1.20	97.00	3.00	0.10	99.90	
	99.60	*0.00*	*0.40*	*99.90*	*0.10*	*99.60*	*0.40*	*0.00*	*100.00*	

Notes: Italicized values refer to $n = 100$.

Table 6

Percentile Values for Selected Distributions with Fixed Mean and Variance

LEIG(θ_1, θ_2)					LN(μ, σ)				
Mean = 0.91315 and Variance = 0.16616									
$\theta_1 = 1.0; \theta_2 = 0.5$					$\mu = 0.8338; \sigma = 0.4263$				
P_{25}	P_{50}	P_{75}	P_{95}	P_{99}	P_{25}	P_{50}	P_{75}	P_{95}	P_{99}
0.6162	0.8221	1.1154	1.7094	2.2325	0.6254	0.8338	1.1115	1.6810	2.2477
Mean = 1.0 and Variance = 1.0									
$\theta_1 = 1.0; \theta_2 = 1.0$					$\mu = 0.7071; \sigma = 0.8326$				
0.3797	0.6758	1.2441	2.9221	4.9841	0.4033	0.7071	1.2398	2.7813	4.9053
Mean = 2.0 and Variance = 33.0									
$\theta_1 = 1.0; \theta_2 = 2.0$					$\mu = 0.6576; \sigma = 1.4915$				
0.1442	0.4568	1.5477	8.5385	24.8412	0.2405	0.6576	1.7982	7.6454	21.1269

G(α, γ)					W(λ, β)				
Mean = 0.91315 and Variance = 0.16616									
$\alpha = 5.0184; \gamma = 0.1820$					$\lambda = 1.0302; \beta = 2.3846$				
P_{25}	P_{50}	P_{75}	P_{95}	P_{99}	P_{25}	P_{50}	P_{75}	P_{95}	P_{99}
0.6159	0.8535	1.1458	1.6706	2.1172	0.6110	0.8834	1.1814	1.6321	1.9546
Mean = 1.0 and Variance = 1.0									
$\alpha = 1.0; \gamma = 1.0$					$\lambda = 1.0; \beta = 1.0$				
0.2877	0.6932	1.3863	2.9957	4.6052	0.2877	0.6932	1.3863	2.9957	4.6052
Mean = 2.0 and Variance = 33.0									
$\alpha = 0.1212; \gamma = 16.5000$					$\lambda = 0.6955; \beta = 0.4226$				
0.0001	0.0335	1.0001	11.3970	28.7995	0.0365	0.2922	1.5065	9.3292	25.8069

5 Illustrative Examples

We first consider a well-known data set from Hogg and Klugman (1984, p. 128) on hurricane losses. This data set consists of 38 observations on losses that exceeded \$1,000,000 for the period 1949 to 1980 as compiled by the American Insurance Association. With censoring below \$5,000,000, using the remaining 35 observations, Hogg and Klugman concluded that the Weibull distribution fits the data best when compared with the lognormal and Pareto distributions, using the Chi-squared goodness-of-fit test. Our second data set is obtained from Klugman, Panjer, and Willmot (1998, Table 1.1, p. 18). This data set corresponds to insurance liability payments and reflects a real-life problem encountered by the authors. The third data set of 96 individual claims is from Currie (1992, Table 1, p. 3). Currie (1992) used the chi-square goodness-of-fit test and concluded that the Pareto model is the best model for this data set.

For these data sets, the parameter estimates and the computed values of the selection statistics, S_i and AIC, for the competing distributions are reported in Table 7. For data sets one and two, both the statistics, S_i and AIC, selected the log-EIG distribution as the underlying distribution that generated the data. For the third data set, while the lognormal was chosen, the log-EIG was the closest competitor among the other families of distributions considered in this study.

6 Concluding Remarks

In this study we consider a recently introduced lifetime distribution, the log-EIG distribution. We show that it has a heavier tail than the gamma or Weibull distributions over certain parameter space. Further, the log-EIG distribution appears to be distinct from the other commonly used lifetime distributions. The illustrative examples indicate the usefulness of the log-EIG distribution in fitting some insurance loss data. In the simulated samples, we observed that the log-EIG distribution generated a few unusually large observations more frequently than the other competing distributions. This feature makes the log-EIG distribution a potentially useful model for insurance claims where extreme observations are not uncommon, such as catastrophic losses in liability claims. Another area where log-EIG can be potentially useful is in lifetime and reliability modeling.

Table 7
Parameter Estimates and Values of the Selection Statistics for Selected Data Sets

Data Set	LEIG		LN	
	θ_1	θ_2	μ	σ
1	43190.8773	1.8970	21587.3367	2.7043
	QK	AIC	QK	AIC
	-451.2414	906.1591	-452.4587	908.1124
	θ_1	θ_2	μ	σ
2	321370.7325	2.8030	113498.2855	6.0245
	QK	AIC	QK	AIC
	-585.0090	1173.5732	-586.2208	1176.3376
	θ_1	θ_2	μ	σ
3	2037.6606	1.7378	1120.4416	1.9565
	QK	AIC	QK	AIC
	-845.5107	1693.3413	-843.5242	1688.9931

Data Set	G		W	
	α	γ	λ	β
1	154916.4796	0.4985	49891.5848	0.6185
	QK	AIC	QK	AIC
	-457.9213	918.7478	-455.5948	914.3550
	α	γ	λ	β
2	4531919.5872	0.2856	400863.9222	0.4153
	QK	AIC	QK	AIC
	-593.5978	1190.5629	-589.8482	1183.5582
	α	γ	λ	β
3	4778.3910	0.6257	2244.5103	0.7132
	QK	AIC	QK	AIC
	-856.9187	1715.5828	-851.1017	1704.1546

†QK - Quesenberry and Kent Criterion; AIC - Akaike Criterion

The selection criterion employed here is suboptimal invariant and it is applicable for uncensored data. The procedure requires that the unknown shape parameter be replaced by a scale invariant estimate. From the results reported in the simulation study, it is clear that this procedure performs well in identifying the true family of distribution that generates a given set of data.

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